Bayesian inference for material parameter identification

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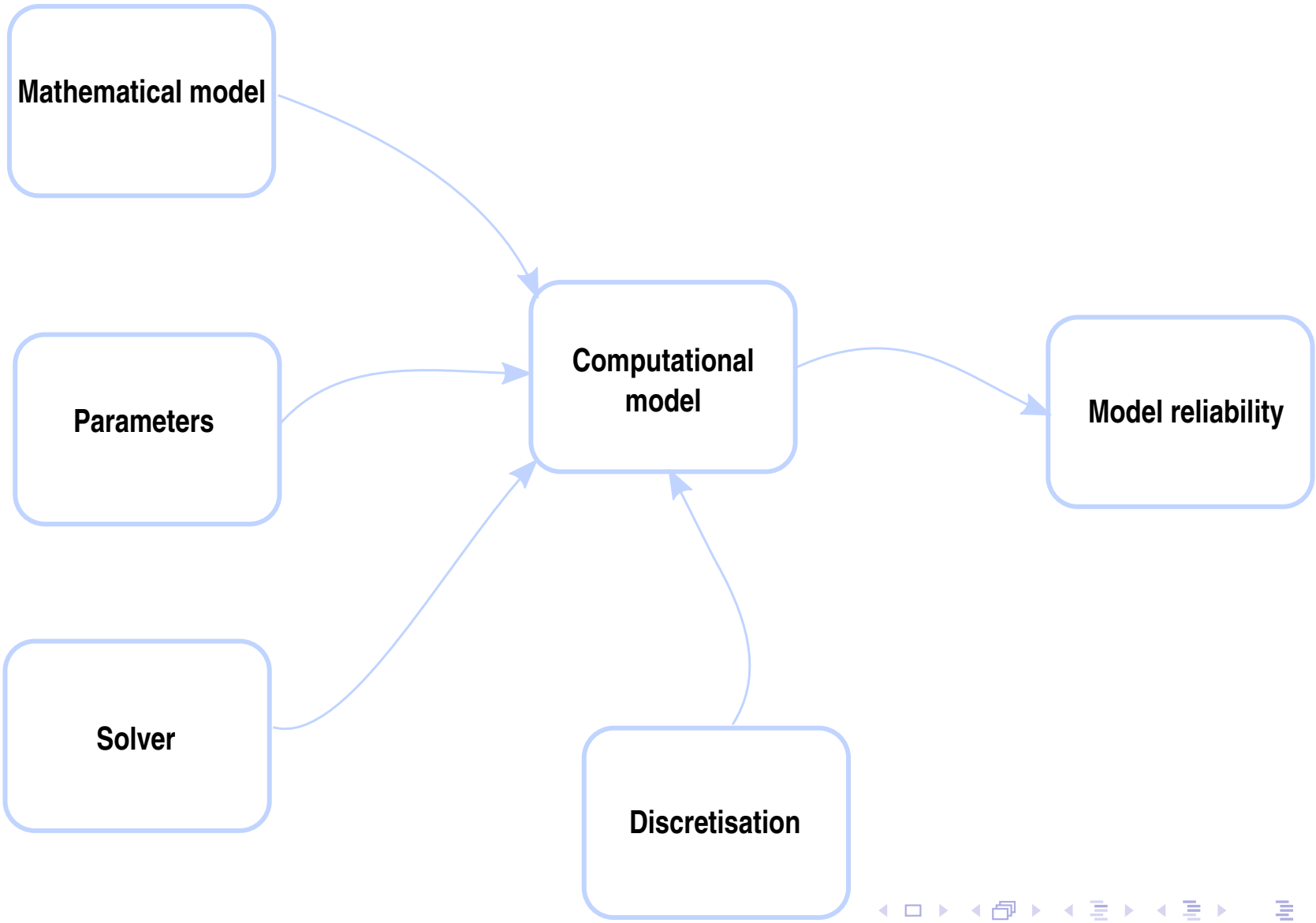
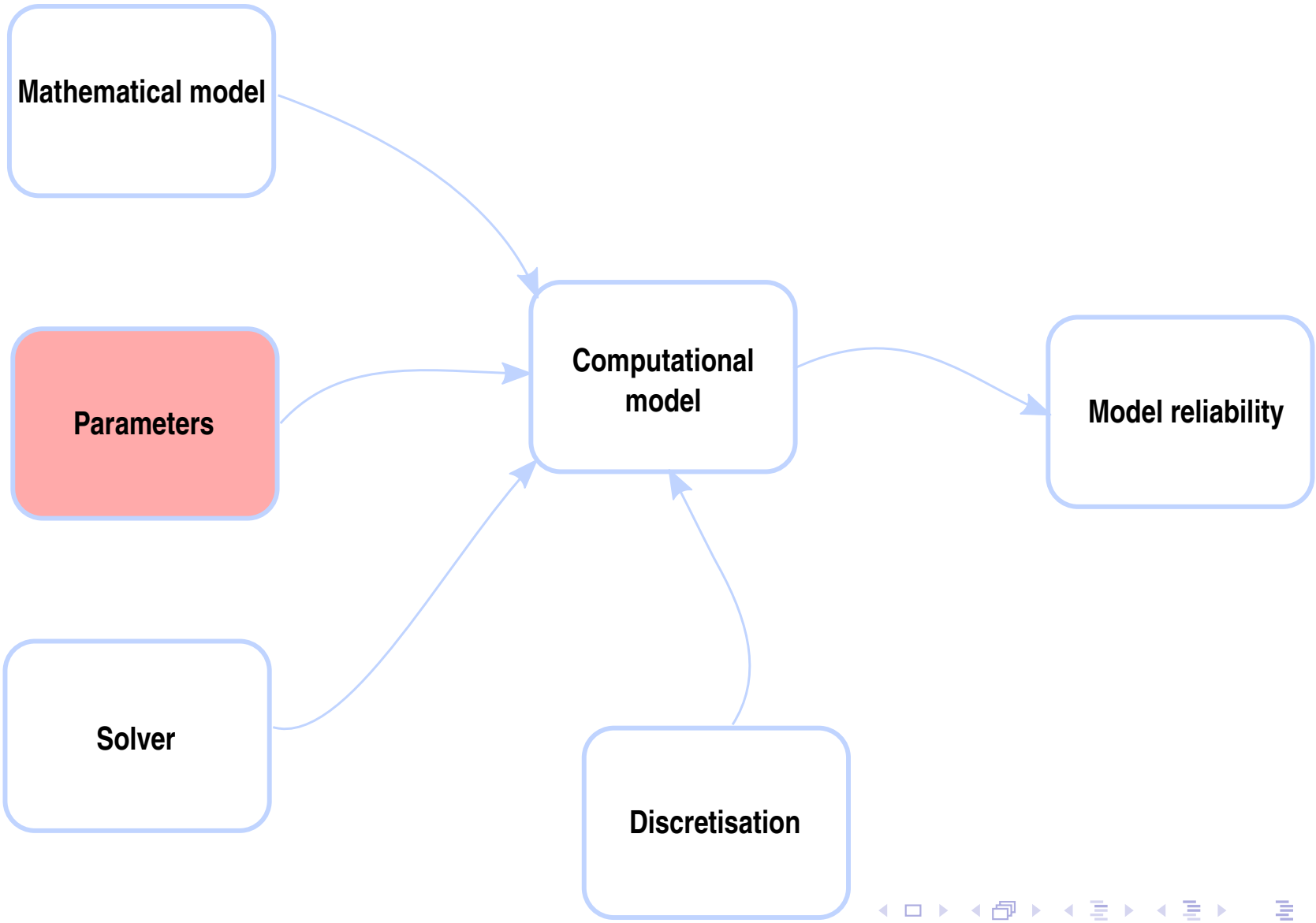
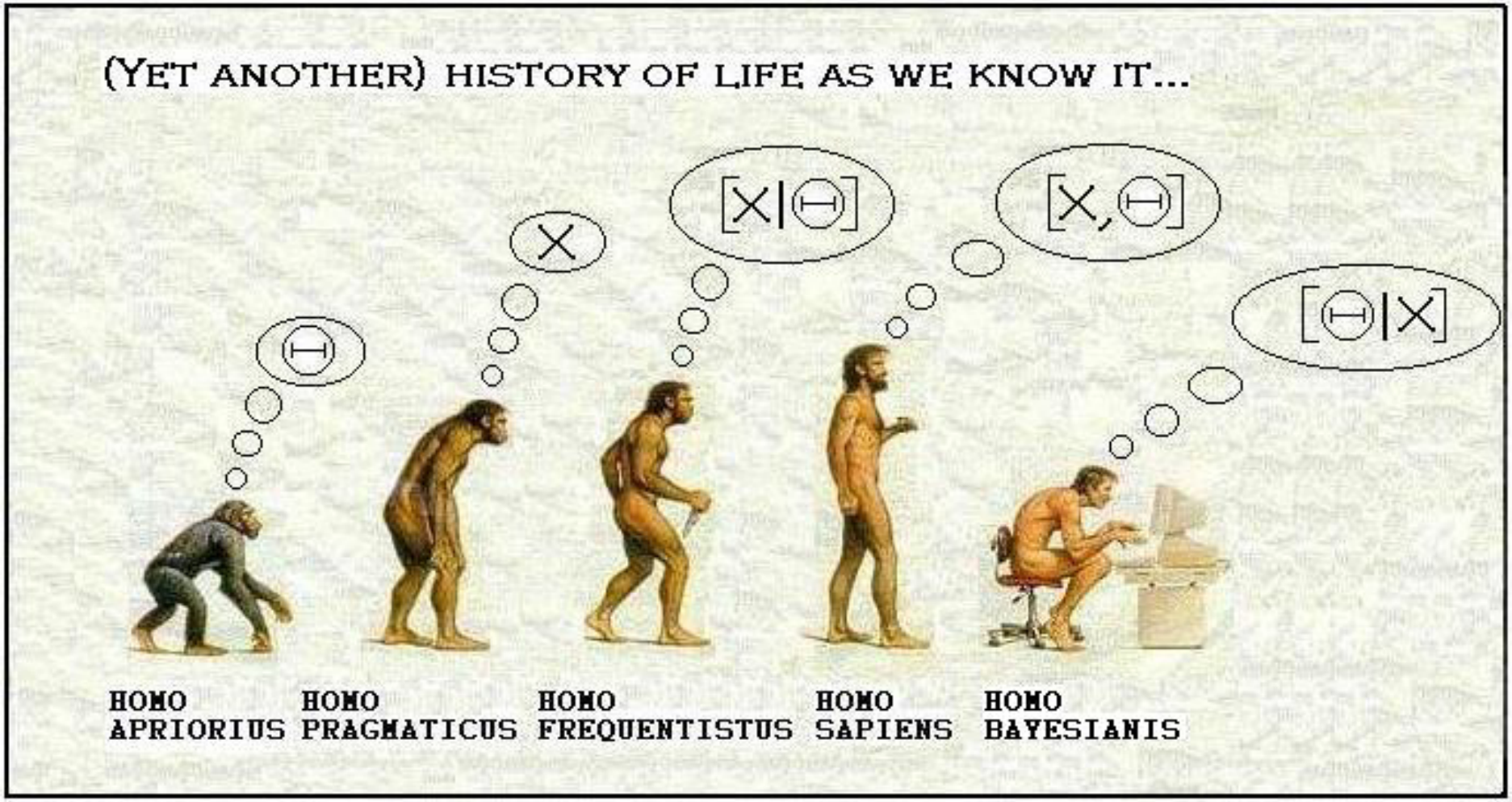
Introduction



Introduction



Identification approaches



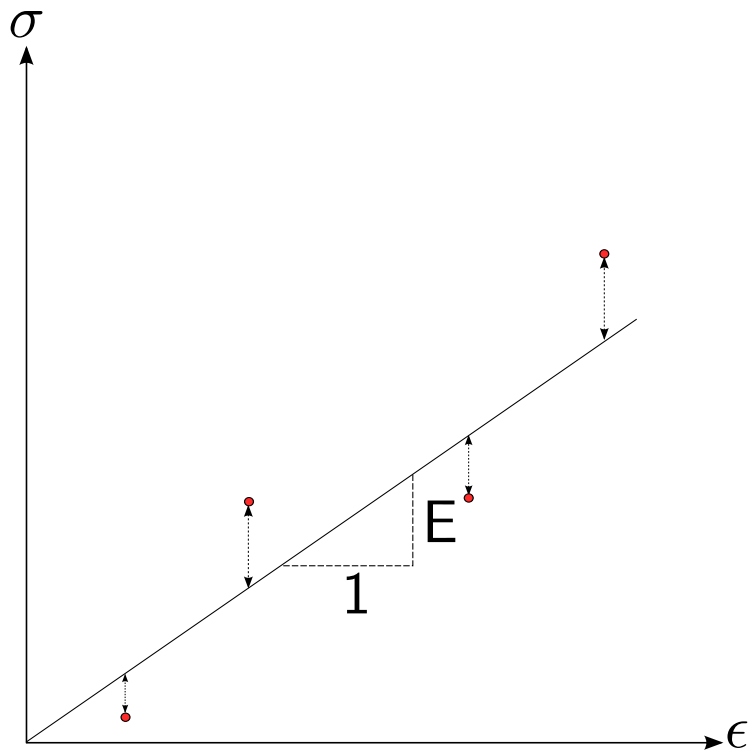
Error minimisation

Least squares method(conventional approach):

$$\sigma = E\epsilon$$

$$J = \frac{1}{2}\sum_{i=1}^{N}(\sigma_i - E\epsilon_i)^2$$

$$\overline{E} = \underset{E}{\operatorname{argmin}} J(E)$$



Frequentist inference

Frequentist inference

10

6

Frequentist inference

$Pr(head) = \frac{10}{16}$

$Pr(tail) = \frac{6}{16}$

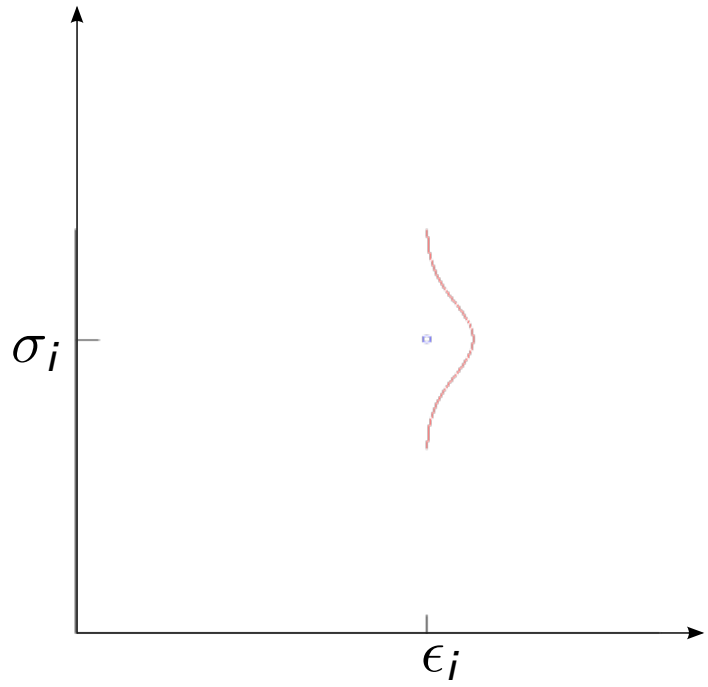
Frequentist inference: Young's modulus identification

Method of maximum likelihood (ML):

$\sigma_i = E\epsilon_i + \Omega$ with

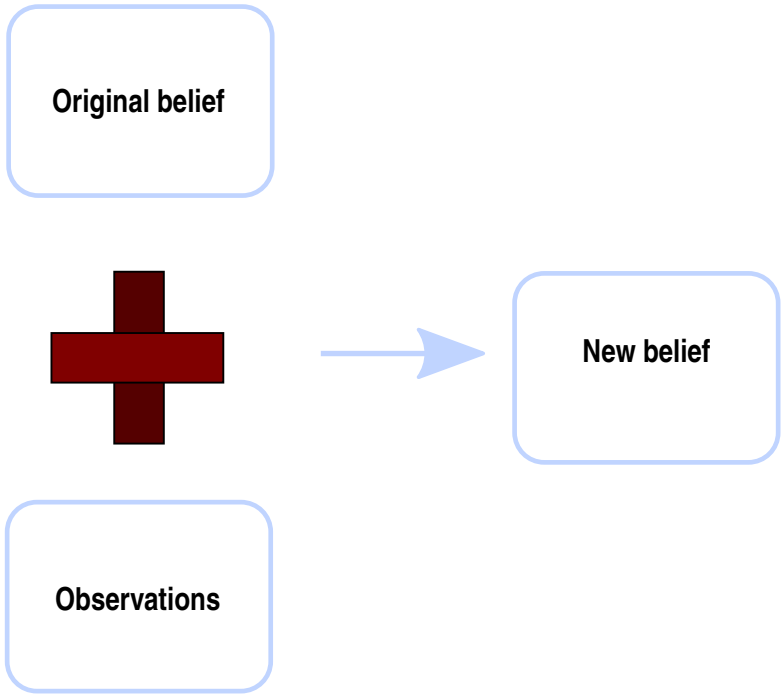
$\pi_{noise}(\omega) = \frac{1}{\sqrt{2\pi}S_{noise}} \exp\Big(- \frac{\omega^2}{2S^2_{noise}} \Big)$

$$\pi(\sigma_i | E, S_{noise}) = \frac{1}{\sqrt{2\pi}S_{noise}} \exp\Big(- \frac{(\sigma_i - E\epsilon_i)^2}{2S^2_{noise}} \Big)$$



Bayesian inference

Bayesian inference



$$\pi(cause|effect) = \frac{\overbrace{\pi(cause)}^{prior} \times \overbrace{\pi(effect|cause)}^{likelihood}}{\underbrace{\pi(effect)}_{evidence}}$$

Bayesian inference: Young's modulus identification

Bayes' formula:

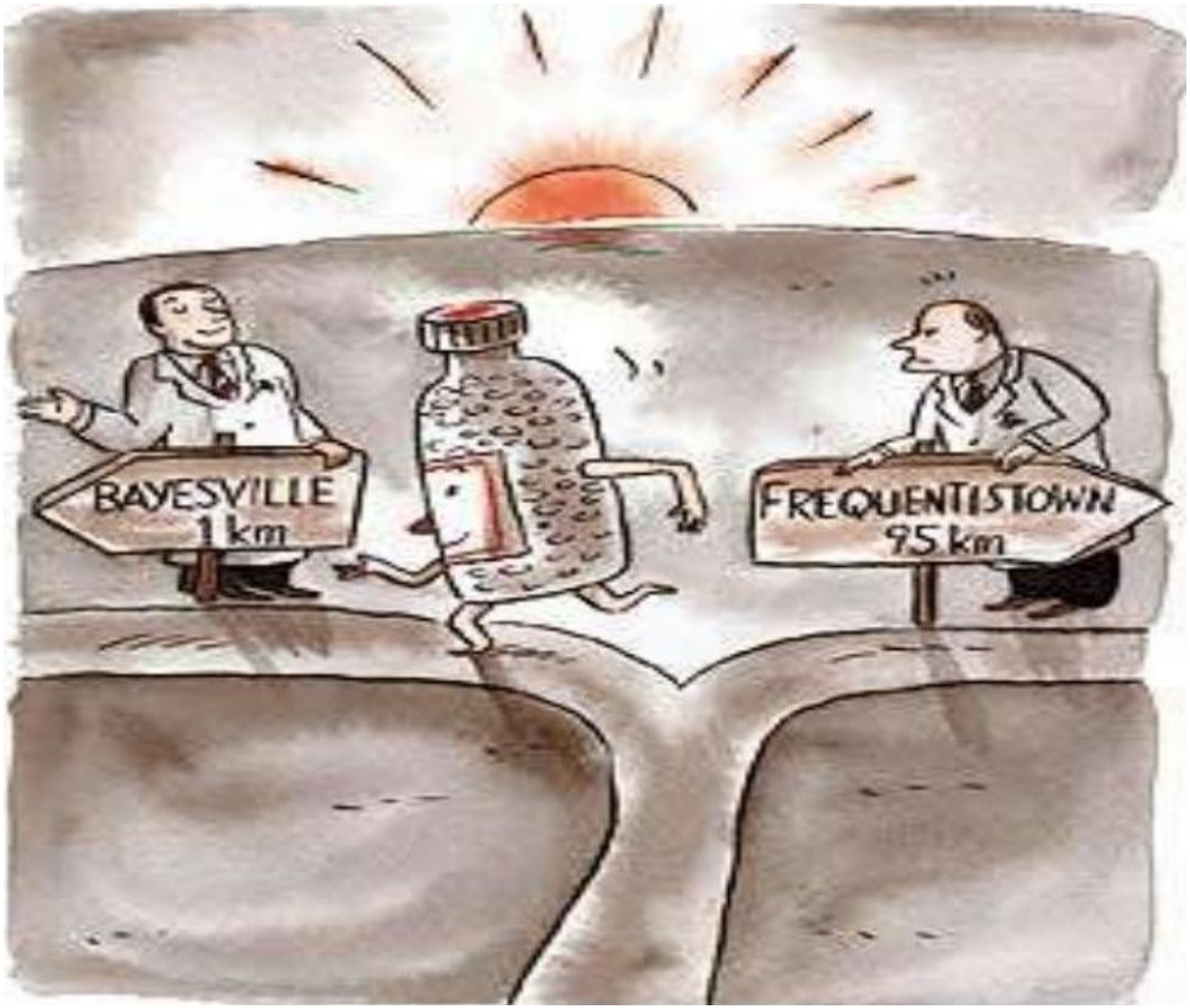
$\sigma_i = E\epsilon_i + \Omega$

Ω : noise in stress measurement

$$\pi(E|\sigma_i) = \frac{\pi(E)\pi(\sigma_i|E)}{\pi(\sigma_i)} \implies \pi(E|\sigma_i) = \frac{\pi(E)\pi(\sigma_i|E)}{C}$$

$$\boxed{\pi(E|\sigma_i) \propto \pi(E)\pi(\sigma_i|E)}$$

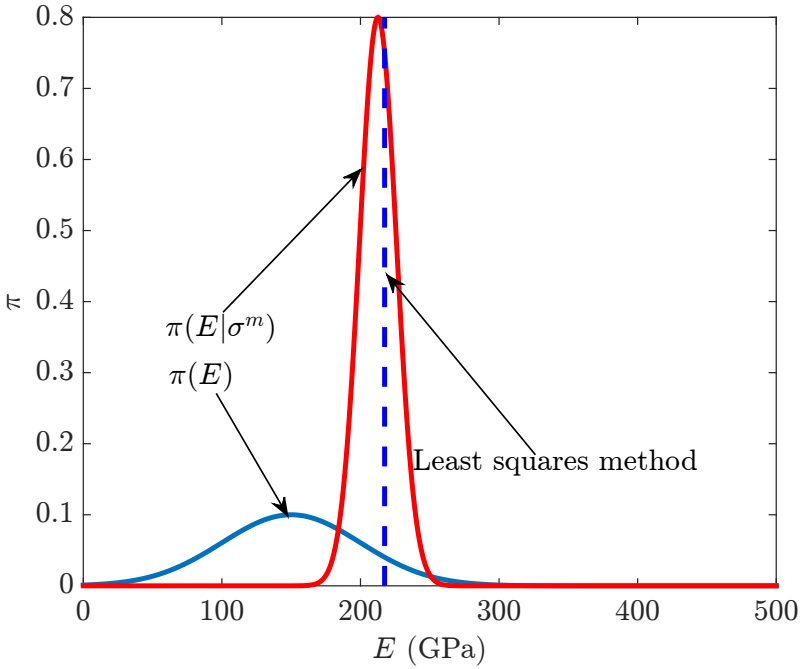
Frequentist vs Bayesian

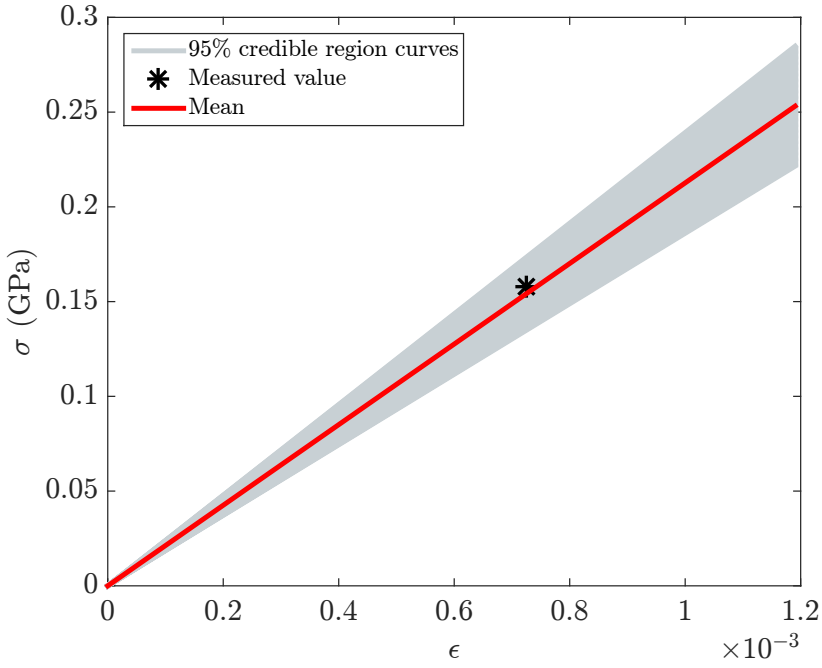


Why do we pick BI?

- BI treats with the parameters as random variables
- You probably will not test hundreds of specimens and then the prior (π_{prior}) may have a positive influence
- For inverse problems, the prior (π_{prior}) regularises the system (avoids ill-posedness)

What have we accomplished?





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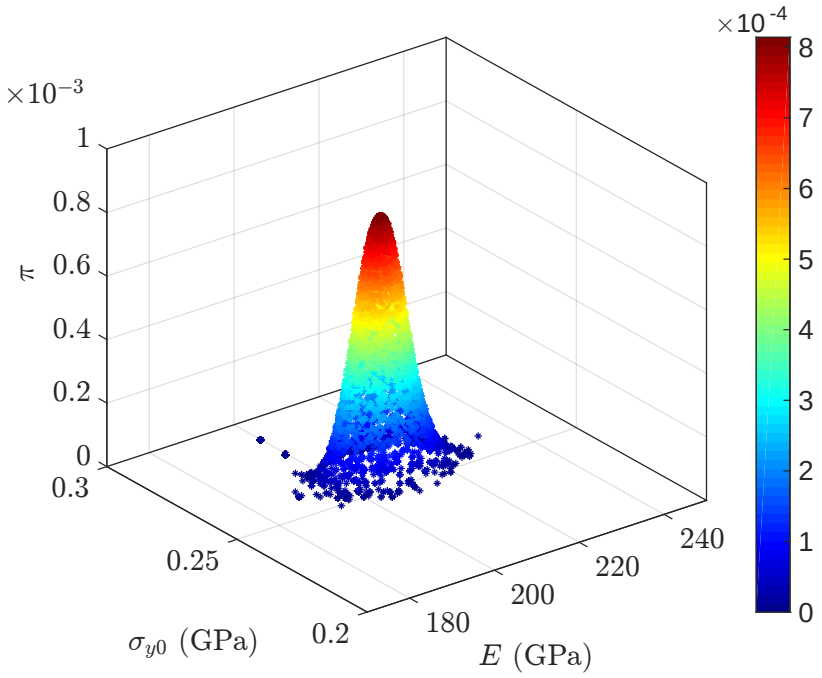

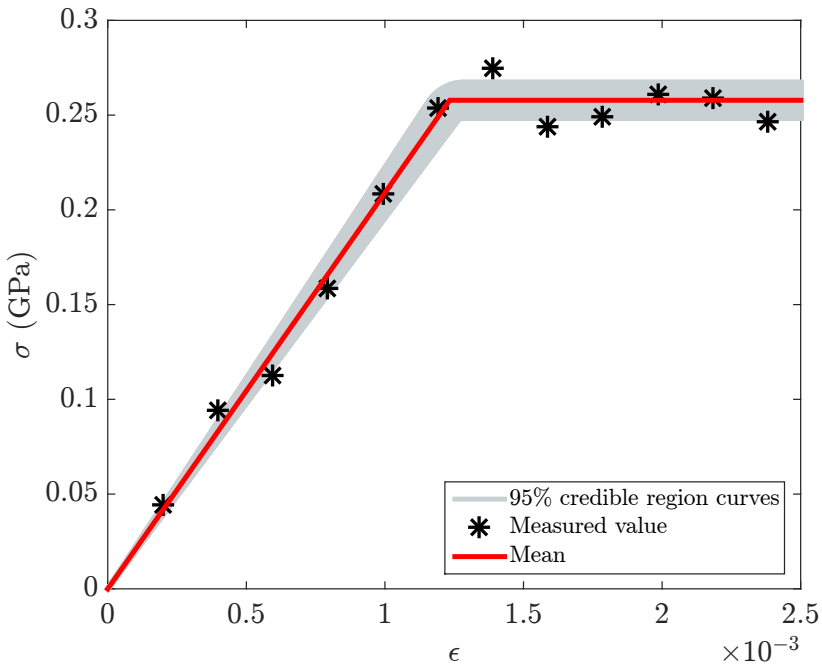

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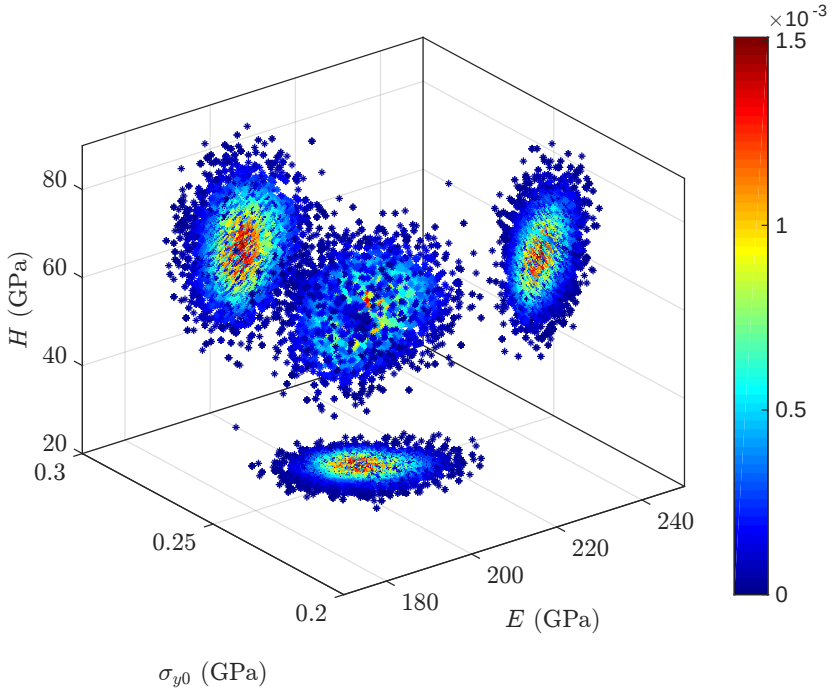

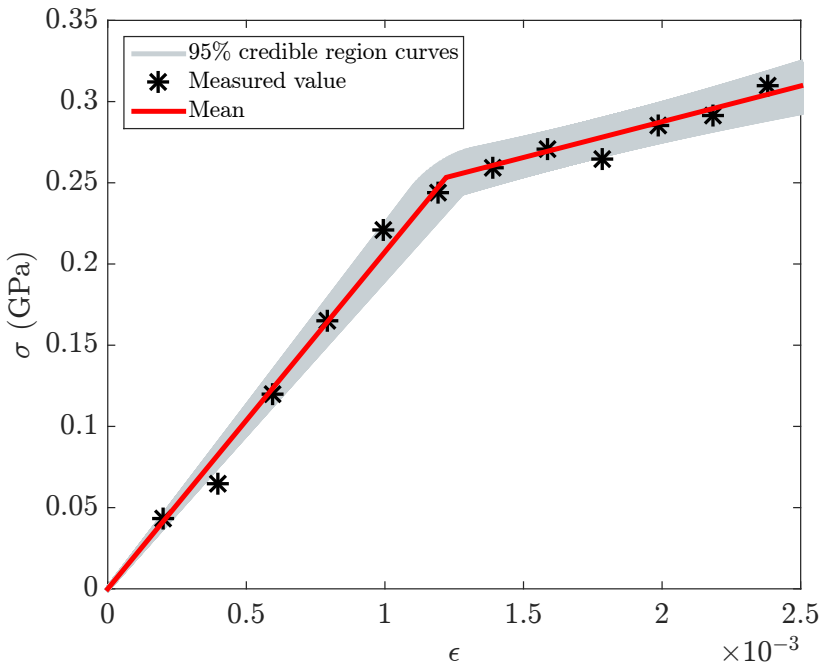

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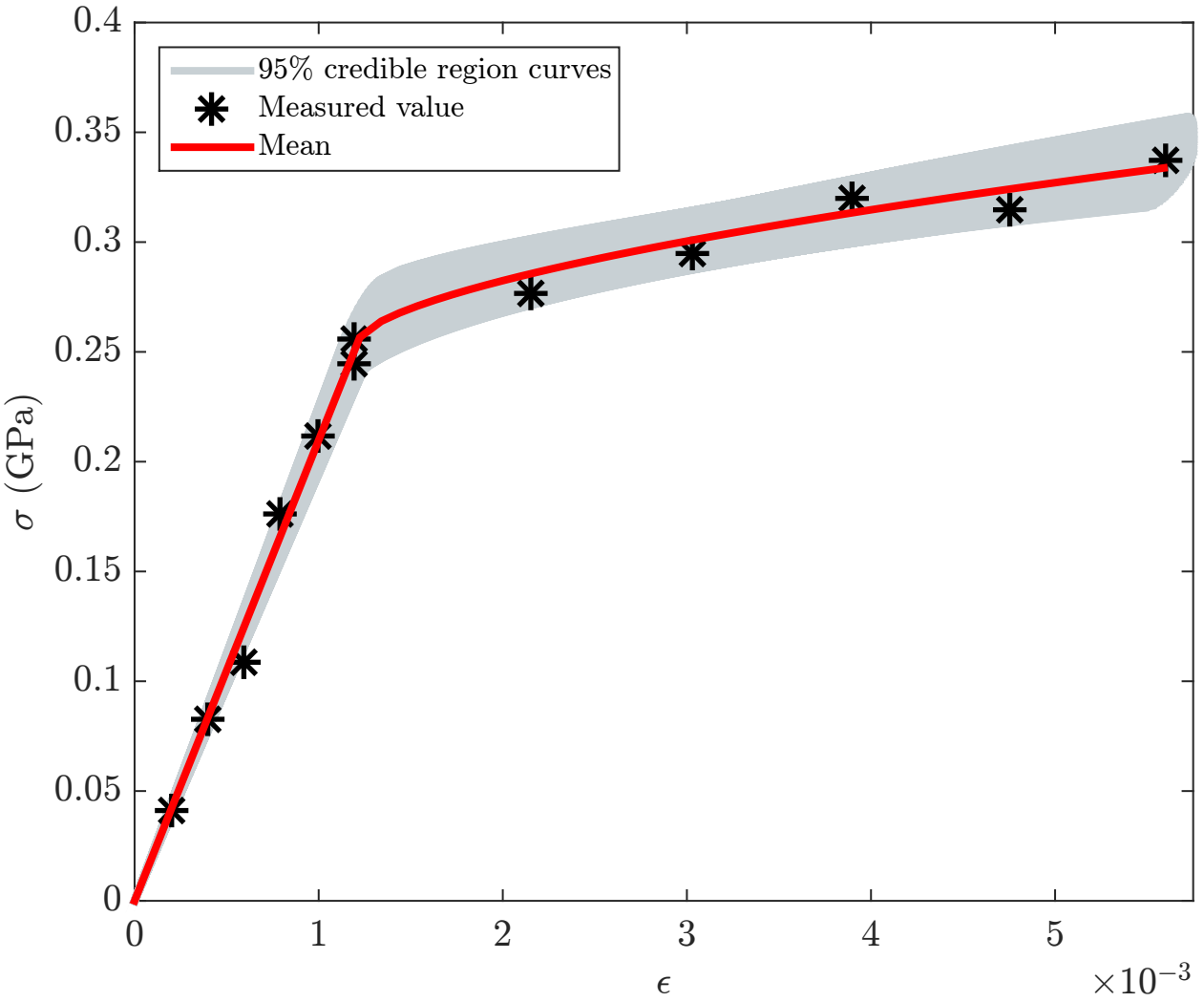

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